High precision measurement of the form factors of the semileptonic decays $K^\pm \rightarrow \pi^0 l^\pm \nu_l \ (K_{l3})$

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On behalf of the NA48/2 Collaboration

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Motivation
- Semileptonic Kaon Decays
- $K_{l3}$ Form Factors
- Form Factor Parametrizations

The NA48/2 Experiment

$K_{l3}$ Form Factor Analysis
- Signal
- Background
- Results
- Summary
Motivation

- Unitarity of the CKM quark mixing matrix can be tested experimentally
- It is an important tool for exploring the limits of the Standard Model
- Any observed deviation from the matrix’s unitarity would either undermine the validity of the Standard Model or indicate existence of a fourth generation of fermions. One of such unitarity constraints is:

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM} \]

- \( \Delta_{CKM} \) parametrizes any possible deviations from the Standard Model induced by new physics operators
- Whole uncertainty of the unitarity relation is dominated by uncertainty on |\( V_{us} \) |
$K^\pm \to \pi^0 l^\pm \nu_l$ decays provide the most accurate and theoretical cleanest way to access $|V_{us}|$

The master formula for $K_{l3}$ decay rates is:

$$\Gamma(K_{l3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+(0)|^2 I^l_K (\lambda_0) (1 + \delta^l_{SU(2)}) + \delta^l_{EM})^2$$

**Experimental Inputs**

- $\Gamma(K_{l3(\gamma)})$ Branching rations and Kaon lifetime
- $I^l_K (\lambda_0)$ Integral of form factor slopes over phase space

**Theoretical Inputs**

- $S_{EW}$ Universal short distance Electro-Weak corrections
- $f_+(0)$ Vector form factor at zero momentum transfer
- $\delta^l_{SU(2)}$ Form factor correction for isospin breaking
- $\delta^l_{EM}$ Long distance Electro-Magnetic effects
$K_{l3}$ From Factors

$K_{l3}$ Decays are described by two form factors $f^+(t)$ and $f^-(t)$ and matrix element can be expressed as:

$$M = \frac{G_F}{2} V_{us} (f^+(t)(P_K + P_{\pi})^\mu \bar{u}_l \gamma_\mu (1 + \gamma_5) \bar{u}_\nu + f^-(t)m_l \bar{u}_l (1 + \gamma_5) \bar{u}_\nu)$$

$t = q^2$ is the squared four momentum transfer to the lepton neutrino system.

$f^-(t)$ can only be measured in the $K_{\mu3}$ decay, because of $m_e \ll m_K$.

$f^\pm(t)$ is the vector form factor and $f^0(t)$ the scalar form factor, they are related by:

$$f^0(t) = f^+(t) + \frac{t}{m_K^2 - m_\pi^2} f^-(t)$$

$f^+(0)$ can not be measured directly, to be given by theory (lattice QCD, $\chi$PT).

Both scalar and vector form factors are measured by experiment relative to $f^+(0)$:

$$\bar{f}^+(t) = \frac{f^+(t)}{f^+(0)} \quad \bar{f}^0(t) = \frac{f^0(t)}{f^+(0)} \quad f^+(0) = f^0(0)$$
Several ways are known to parametrize the $K_{l3}$ form factors.

**Pole Parametrization:** One single resonance (vector (1-) or scalar (0+)) is assumed to be responsible for the process. The corresponding masses $m_V$ and $m_S$ are only free parameters and can be estimated from the fit:

$$\bar{f}_{+,0}(t) = \frac{m_{V,S}}{m_{V,S}^2 - t}$$

**Taylor expansion** Linear/quadratic expansion in the momentum transfer $t = q^2$

$$\bar{f}_{+,0}(t) = \left[1 + \lambda_{+,0} \frac{t}{m^2}\right] \quad \text{Linear}$$

$$\bar{f}_{+,0}(t) = \left[1 + \lambda'_{+,0} \frac{t}{m^2} + \frac{\lambda''_{+,0}}{2} \left(\frac{t}{m^2}\right)^2\right] \quad \text{Quadratic}$$

More free parameters to be determined $\implies$ Strong correlation between parameters.

Other parametrizations (Dispersive, Z-fit) are also known, but not yet used in this analysis.
NA48/2 is a fixed target experiment in the North Area of CERN the SPS.

Main purpose was the search of **direct CP violation** in the $K_{3\pi}$ by measuring charge asymmetry in charged Kaon decays:

\[
\begin{align*}
    K^\pm &\to \pi^\pm \pi^0 \pi^0 \\
    K^\pm &\to \pi^\pm \pi^+ \pi^-
\end{align*}
\]
Simultaneous $K^+$ and $K^-$ beams with $p_K = (60 \pm 1.8)$ GeV/c

3 days of dedicated run with minimum trigger requirements for the $K_{l3}$ measurement
Main Sub-Detectors:

- **Magnetic spectrometer**
  4 sets of drift chambers: $\Delta p/p \approx 0.45\%$ for typical momentum of 20 GeV/c

- **Charged and Neutral Hodoscopes**
  $\sigma_t = 150$ ps

- **Liquid Krypton Calorimeter**
  $\Delta E/E = 1.0\%$ GeV/c for typical energy of 20 GeV

- **Hadron calorimeter, photon vetos, muon counters**

**Trigger Condition:**
Hodoscope space point + $E_{Lkr} > 10$ GeV
**$K_{l3}$ Event Selection**

- **1 good track**
  Muon identification using muon counter and $E/p$
  Electron identification using $E/p$

- **1 good $\pi^0 \rightarrow \gamma \gamma$**
  Pion mass cut: $|m_{\gamma\gamma} - m_{\pi^0}^{PDG}| < 10 \text{ MeV}^2$

- **Event reconstruction**
  Lkr clusters and muon track consistent in time.
  Missing mass cut using calculation with $K_{l3}$ hypothesis
  $$(M_{miss})_{Kl3}^2 = (P_K - P_l - P_{\pi^0})^2 < 10 \text{ MeV}^2$$
  Kaon energy reconstruction under the assumption of a missing undetected neutrino within the range of:
  $55 \text{ GeV} < E_{K^\pm} < 65 \text{ GeV}$

- **$2.5 \times 10^6 K_{\mu3}^\pm$** events selected
- **$4.0 \times 10^6 K_{e3}^\pm$** events selected
$K^\pm \rightarrow \pi^\pm \pi^0 \,(K_{2\pi^0})$ Background rejection

$K^\pm_{\mu 3}:

\textbf{Main Background: } K^\pm \rightarrow \pi^\pm \pi^0 \text{ with } \pi^\pm \rightarrow \mu^\pm \text{ mis-ID can fake signal:}

- Without suppression, $K^\pm \rightarrow \pi^\pm \pi^0$ bkg at the level of 20%

- Cut in the invariant $\pi^\pm \pi^0$ - mass and the transverse momentum of the pion:
  - Background contamination reduced to 0.5%.
  - about 24% loss of $K^\pm_{\mu 3}$ events.
$K^\pm \rightarrow \pi^\pm \pi^0 \ (K_{2\pi^0})$  

**Background rejection**

\[ K_{e3}^\pm : \]

**Main Background:** $K^\pm \rightarrow \pi^\pm \pi^0$ with $\pi^\pm \rightarrow e^\pm$ mis-ID can fake signal:

- $\pi^\pm$ with $E/p > 0.95$ can fake a $K_{e3}$ decay

- Cut in transverse momentum of the event
  - Background contamination reduced $< 0.5\%$.
  - $K_{e3}^\pm$ acceptance loss $\approx 3\%$.
$K^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0 (K_{3\pi^0})$ Background

$K_{\mu3}^{\pm}$:

- $\pi^{\pm} \rightarrow \mu^{\pm}$ decay with one lost photons from $\pi^0$ decays.
- Small but introduces slope in the Dalitz plot.
- No dedicated cut to reduce the background.
- A correction is applied to take the background into account.
- Without the correction result shifts by $\approx 0.5 \sigma_{\text{stat}}$.

$K_{e3}^{\pm}$:

- $K^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0$ is negligible.
$K_{\mu 3}^\pm$:  

$K_{l3}$ decay rate including first order radiative corrections can be written as:

$$\Gamma_{K_{l3}} = \Gamma^0_{K_{l3}} + \Gamma^1_{K_{l3}} = \Gamma^0_{K_{l3}} (1 + 2\delta_{K_{l3}}^{EM})$$


<table>
<thead>
<tr>
<th>Mode</th>
<th>$\delta K_{EM}^{\mu 3}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu 3}^0$</td>
<td>$0.700 \pm 0.110$</td>
</tr>
<tr>
<td>$K_{\mu 3}^\pm$</td>
<td>$0.008 \pm 0.125$</td>
</tr>
</tbody>
</table>

For $K_{\mu 3}^\pm$ small effect on the acceptance.

$\sim 1\%$ effect on the Dalitz plot slope.
$K_{e3}^{\pm}$:

$K_{l3}$ decay rate including first order radiative corrections can be written as:

$$\Gamma_{K_{l3}} = \Gamma_{K_{l3}}^0 + \Gamma_{K_{l3}}^1 = \Gamma_{K_{l3}}^0 (1 + 2\delta_{EM}^{K_{l}})$$


<table>
<thead>
<tr>
<th>Mode</th>
<th>$\delta K_{e3}^{EM}$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{e3}^0$</td>
<td>0.495 ± 0.100</td>
</tr>
<tr>
<td>$K_{e3}^\pm$</td>
<td>0.495 ± 0.100</td>
</tr>
</tbody>
</table>

For $K_{e3}^{\pm}$ the effect on the acceptance are bigger.

$\sim 10\%$ effect on the Dalitz plot slope.
**Form Factor Fitting Procedure**

To extract form factors, a fit to the Dalitz plot density is applied

\[
\rho(E^*_l, E^*_\pi) = \frac{d^2N(E^*_l, E^*_\pi)}{dE^*_\mu dE^*_\pi} \propto Af_+^2(t) + Bf_+(t)(f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} + C \left[(f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t}\right]^2
\]

\(E^*_l\) and \(E^*_\pi\) are the energy of the lepton and the pion in the Kaon rest frame.

Cells which are outside or crossing the border of the physical region of Dalitz plot are not used in the fit. The fit is performed on cells of \(5 \times 5\) MeV\(^2\).

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**Data-MC Comparison**

- **Pion energy in the kaon rest frame:** $K_{\mu 3}$
- **Pion energy in the kaon rest frame:** $K_{e 3}$

$\Rightarrow$ Data-MC difference mostly below the 0.1% level.

$\Rightarrow$ Remaining differences taken into account by systematics.
Study of Systematic Errors

<table>
<thead>
<tr>
<th>$K_{\mu 3}^\pm$</th>
<th>$\Delta \lambda'_+ \times 10^{-3}$</th>
<th>$\Delta \lambda''_+ \times 10^{-3}$</th>
<th>$\Delta \lambda_0$</th>
<th>$\Delta m_V$</th>
<th>$\Delta m_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaon Energy</td>
<td>±0.1</td>
<td>±0.0</td>
<td>±0.3</td>
<td>±1</td>
<td>±8</td>
</tr>
<tr>
<td>Vertex</td>
<td>±1.0</td>
<td>±0.5</td>
<td>±0.1</td>
<td>±2</td>
<td>±7</td>
</tr>
<tr>
<td>Bin size</td>
<td>±0.8</td>
<td>±0.4</td>
<td>±0.7</td>
<td>±3</td>
<td>±10</td>
</tr>
<tr>
<td>Energy scale</td>
<td>±0.3</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0</td>
<td>±1</td>
</tr>
<tr>
<td>Acceptance</td>
<td>±0.2</td>
<td>±0.1</td>
<td>±0.3</td>
<td>±2</td>
<td>±5</td>
</tr>
<tr>
<td>$K_{2\pi}$ background</td>
<td>±1.7</td>
<td>±0.5</td>
<td>±0.6</td>
<td>±3</td>
<td>±0</td>
</tr>
<tr>
<td>2nd Analysis</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0.2</td>
<td>±2</td>
<td>±5</td>
</tr>
<tr>
<td>FF input</td>
<td>±0.3</td>
<td>±0.8</td>
<td>±0.1</td>
<td>±7</td>
<td>±3</td>
</tr>
<tr>
<td>Systematic</td>
<td>±2.2</td>
<td>±1.1</td>
<td>±1.0</td>
<td>±9</td>
<td>±16</td>
</tr>
<tr>
<td>Statistical</td>
<td>±3.0</td>
<td>±1.1</td>
<td>±1.4</td>
<td>±8</td>
<td>±31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$K_{e 3}^\pm$</th>
<th>$\Delta \lambda'_+ \times 10^{-3}$</th>
<th>$\Delta \lambda''_+ \times 10^{-3}$</th>
<th>$\Delta m_V$ MeV/c^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaon Energy</td>
<td>±0.3</td>
<td>±0.1</td>
<td>±6</td>
</tr>
<tr>
<td>Vertex</td>
<td>±0.2</td>
<td>±0.1</td>
<td>±0</td>
</tr>
<tr>
<td>Bin size</td>
<td>±0.0</td>
<td>±0.1</td>
<td>±2</td>
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<tr>
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<td>±0.3</td>
<td>±3</td>
</tr>
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$\Rightarrow K_{\mu 3}^\pm$ is dominated by statistics.
Main error from $K_{2\pi}$ background.

$\Rightarrow K_{e 3}^\pm$ is dominated by systematics.
# Preliminary Results for the NA48/2

## Outline
- **Motivation**
  - The NA48/2 Experiment
  - $K_{l3}^+$ Form Factor Analysis
- **Signal**
- **Background**
- **Radiative Corrections**
- **Form Factor extraction**
- **Results**
- **Systematics**
- **Summary**

## Preliminary Results for the NA48/2

### Table

<table>
<thead>
<tr>
<th>Quadratic ($\times 10^{-3}$)</th>
<th>$\lambda_+''$</th>
<th>$\lambda_+''$</th>
<th>$\lambda_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu3}^+$</td>
<td>26.3 ± 3.0$<em>{\text{stat}}$ ± 2.2$</em>{\text{syst}}$</td>
<td>1.2 ± 1.1$<em>{\text{stat}}$ ± 1.1$</em>{\text{syst}}$</td>
<td>15.7 ± 1.4$<em>{\text{stat}}$ ± 1.0$</em>{\text{syst}}$</td>
</tr>
<tr>
<td>$K_{e3}^+$</td>
<td>27.2 ± 0.7$<em>{\text{stat}}$ ± 1.1$</em>{\text{syst}}$</td>
<td>0.7 ± 0.3$<em>{\text{stat}}$ ± 0.4$</em>{\text{syst}}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pole (MeV/c$^2$)</th>
<th>$m_V$</th>
<th>$m_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu3}^+$</td>
<td>873 ± 8$<em>{\text{stat}}$ ± 9$</em>{\text{syst}}$</td>
<td>1183 ± 31$<em>{\text{stat}}$ ± 16$</em>{\text{syst}}$</td>
</tr>
<tr>
<td>$K_{e3}^+$</td>
<td>879 ± 3$<em>{\text{stat}}$ ± 7$</em>{\text{syst}}$</td>
<td></td>
</tr>
</tbody>
</table>

## Diagrams

- **68% Confidence level contours**
  - KTeV $K^0$
  - KLOE $K^0$
  - Istra+ $K^-$
  - NA48 $K^0$
  - NA48/2 $K^z$
    - preliminary

- **$K_{e3}^+$**
  - FlaviaNet Fit $K_{e3}$ 2010

- **$K_{\mu3}^+$**
  - FlaviaNet Fit $K_{\mu3}$ 2010
  - FlaviaNet Fit $K_{e3}$ 2010

- **$\lambda_0$**

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High precision measurement of the form factors of the semileptonic decays
Statistical and systematic uncertainties combined:

<table>
<thead>
<tr>
<th>Quadratic ($\times 10^{-3}$)</th>
<th>$\chi'<em>+^{K</em>{\mu3}K_{e3}}$</th>
<th>$\chi''<em>+^{K</em>{\mu3}K_{e3}}$</th>
<th>$\lambda_0^{K_{\mu3}K_{e3}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu3}^+K_{e3}^+$ combined</td>
<td>26.98 $\pm$ 1.11</td>
<td>0.81 $\pm$ 0.46</td>
<td>16.23 $\pm$ 0.95</td>
</tr>
<tr>
<td>Pole (MeV/c$^2$)</td>
<td>$m_V$</td>
<td>$m_S$</td>
<td>$m_S$</td>
</tr>
<tr>
<td>$K_{\mu3}^+K_{e3}^+$ combined</td>
<td>877 $\pm$ 6</td>
<td>1176 $\pm$ 31</td>
<td>1176 $\pm$ 31</td>
</tr>
</tbody>
</table>

$K_{l3}^0$ results from KLOE, KTeV and NA48, $K_{l3}^-$ from ISTRA.

NA48/2 is the first measurement which uses $K_{\mu3}^\pm$ and $K_{e3}^\pm$.

NA48/2 preliminary results have smallest errors.
Summary and Outlook

- 4 millions of $K_{e3}$ and 2.5 millions of $K_{\mu3}$ events analysed
  - First measurement with $K^+$ and $K^-$
  - Very small background
  - Good agreement between $K_{e3}$ and $K_{\mu3}$ results
  - Preliminary results competitive with the world average

- $\sim$10 times larger $K_{e3}$ and $K_{\mu3}$ data-samples are recorded in 2007 by the NA62 experiment. Analysis of $K_{e3}$ data is ongoing. First results are expected soon.

| Quadratic ($\times 10^{-3}$) | $\lambda'_+ | $\lambda''_+ | $\lambda_0$
|-----------------------------|-----------------|-----------------|----------------|
| $K_{\mu3}^\pm K_{e3}^\pm$ combined | $26.98 \pm 1.11$ | $0.81 \pm 0.46$ | $16.23 \pm 0.95$
| Pole (MeV/c^2) | $m_V$ | $m_S$
| $K_{\mu3}^\pm K_{e3}^\pm$ combined | $877 \pm 6$ | $1176 \pm 31$
Thanks for your attention!
Dispersive Parametrization:

Uses dispersive techniques and known low energy $K-\pi$ phases to parametrize vector and scalar form factors:

$$f_+(t) = \exp \left( \frac{t}{m^2_\pi} (\Lambda_+ + H(t)) \right);$$

$$f_0(t) = \exp \left( \frac{t}{m^2_K - m^2_\pi} (\ln(C') - G(t)) \right);$$

$\Lambda_+$ is slope of the vector form factor

$\ln(C') = \ln[f_0(m^2_K - m^2_\pi)]$ is the logarithm of the scalar form factor at the Callan-Treiman point

Accurate polynomial approach for the dispersive integrals $H(t)$ and $G(t)$ are available.