Isospin breaking and $f_0(980) - a_0(980)$ mixing in the $\eta(1405) \rightarrow \pi^0 f_0(980)$ reaction

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introduction

discussion assuming a contact $\eta' \rightarrow \pi^0 PP$ vertex

study of a triangular mechanism first proposed by J. -J. Wu et al.

Results

Conclusion
**BES experiment** → [M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 108, 182001 (2012)]
- very narrow signal for the isospin violating channel → in agreement with previous findings
- very large isospin violation in \( \eta(1405) \rightarrow \pi^0 f_0(980) \) compared to \( \eta(1405) \rightarrow \pi^0 a_0(980) \)
  (in comparison with other reactions such as \( J/\psi \rightarrow \phi \pi^0 \eta(\pi^+ \pi^-) \))

\[
\frac{\Gamma(\pi^0, \pi^0 \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)} = 18\%
\]

→ difficult to explain even assuming a large \( I = 0-I = 1 \) mixture

**starting point** → \( a_0(980) \) and \( f_0(980) \) are dynamically generated by the meson-meson interaction provided by chiral Lagrangians

- building blocks: \( \pi \pi, K \bar{K} \) for \( f_0(980) \) and \( \pi \eta, K \bar{K} \) for \( a_0(980) \)
- success in the study of \( \Phi \rightarrow \pi^0 \pi^0 \gamma, \pi^0 \eta^{[1]} \), \( J/\psi \rightarrow \Phi(\omega) f_0^{[2]} \)... gives support to this assumption
Formalism assuming local primary $\eta(1405) \to \pi^0PP$ vertices

Assumptions:

- $\eta(1405) \to \pi^0PP \implies$ described by contact (or contactlike) vertices

- $\eta(1405)$ considered as an $I = 0$ state $\implies$ PP pair of interacting mesons: $I = 1$

  $I = 1$ combination $\implies \frac{1}{\sqrt{2}}(K^+K^- - K^0\bar{K}^0)$

  $\implies$ the physical masses of the kaons lead to an isospin breaking effect

- $\eta(1405)$ considered as an SU(3) singlet $\implies$ since the $\pi^0$ is an octet, the pair PP must be octet

  using $8 \otimes 8 \to 1$ decomposition $\implies M_{K^+K^-} = \sqrt{\frac{3}{5}}, \ M_{K^0\bar{K}^0} = -\sqrt{\frac{3}{5}}, \ M_{\pi^0\eta} = \sqrt{\frac{4}{5}}$
Formalism assuming local primary $\eta(1405) \rightarrow \pi^0PP$ vertices

Scattering matrix for the production of the final state:

$$t_f = M_f + \sum_{i=1}^{3} M_i G_i T_{if}$$

- $T_{if}$: $5 \times 5$ scattering matrix for the channels $K^+K^-, K^0\bar{K}^0$, $\pi^0\eta$, $\pi^+\pi^-, \pi^0\pi^0$

- $M_i = A \left( \sqrt{\frac{3}{5}}, -\sqrt{\frac{3}{5}}, \sqrt{\frac{4}{5}}, 0, 0 \right)$
  $A =$constant

- $G_i = \int |q| < q_{\text{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1 \omega_2} \frac{1}{p^{02} - (\omega_1 + \omega_2)^2 + i\varepsilon}$
  with $q_{\text{max}} = 900 \text{ MeV}$

$G_i$ and $T_{if} \implies 2$ sources of isospin violation

We compare $\frac{d\Gamma}{dm_f} = \beta p_1 \tilde{p}_2 |t_f|^2$ with the experiment

- $m_f =$ invariant mass of the final pair ($\pi^+\pi^-$ or $\pi^0\eta$)
- $\beta =$ constant

- $p_1 = \frac{\lambda^{1/2}(m_{\eta'}^2, m_\pi^2, m_f^2)}{2m_{\eta'}}$ and $\tilde{p}_2 = \frac{\lambda^{1/2}(m_{\pi^0}^2, m_\pi^2, m_f^2)}{2m_f}$

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![Graph of $d\Gamma/dm_f$ for $\eta' \to \pi^0\pi^+\pi^-$ decay in the $f_0(980)$ region]

- **Very narrow peak**: width $\sim 10$ MeV
  - in agreement with BES results
  - it's not the shape of the $f_0(980)$

![Graph of $d\Gamma/dm_f$ for $\eta' \to \pi^0\pi^0\eta$ decay in the $a_0(980)$ region]

- much larger width
- ratio of strengths at the peak $\sim 3\%$
Results assuming local primary $\eta(1405) \to \pi^0$PP vertices

shape of the peak for $\pi^+\pi^-$ production

$\Rightarrow$ due to the fact that $G_{K^+K^-} - G_{K^0\bar{K}^0}$ is very small away from the two thresholds

$\Rightarrow$ the $K\bar{K}$ thresholds show up as cusps in $(d\Gamma/dm_f)_{\pi^+\pi^-} / (d\Gamma/dm_f)_{\pi^0\eta}$

[L. Roca, Phys. Rev. D 88, 014045 (2013)]

**Ratio** $\left(\frac{d\Gamma}{dm_f}\right)_{\pi^+\pi^-} / \left(\frac{d\Gamma}{dm_f}\right)_{\pi^0\eta}$ as a function of $m_f$

$$\frac{\Gamma(\pi^0, \pi^+\pi^-)}{\Gamma(\pi^0, \pi^0\eta)} = 1.5\%$$

along the lines of 0.6% observed in $J/\psi \to \phi\pi^0\eta(\pi^+\pi^-)$ and $\chi_{c1} \to \pi^0(\pi^+\pi^-)(\pi^0\eta)$

Is the $\eta(1405)$ an SU(3) singlet?

$$R = \frac{M(\pi^0\eta)}{M(K^+K^-)} \Rightarrow \text{its order of magnitude can be determined from experiment}$$

From


$$|R| = 0.75 \pm 0.17$$

with positive sign would be in agreement with the pure singlet assumption $R = \sqrt{4/3} = 1.15$ ($R = 0$ for the decuplet and $R = -\sqrt{3}$ for the 27)

in order to evaluate the uncertainties due to diversion from the SU(3) singlet assumption, we study the results for $R \in [-1, 1.2]$
Is the $\eta(1405)$ an SU(3) singlet?

\[ \frac{\Gamma(\pi^0, \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)} \in [1\% - 4\%] \]

\[ \Rightarrow \text{we cannot obtain the ratio of 18\% found by BES even considering the uncertainties} \]
Is the $\eta(1405)$ an $I = 0$ object?

If the $\eta(1405)$ were an $I = 1$ object:
- interacting pair: $I = 0$ (to magnify the $f_0(980)$ production)
- $K\bar{K}$ $I = 0$ combination $\rightarrow \frac{1}{\sqrt{2}}(K^+K^- + K^0\bar{K}^0)$

Let us consider an isospin mixture $\Rightarrow \tilde{M}_i = A \left( (1 + \alpha)\sqrt{\frac{3}{5}}, (\alpha - 1)\sqrt{\frac{3}{5}}, \sqrt{\frac{4}{5}}, 0, 0 \right)$

with $\alpha = \text{measure of the mixture}$

to get the ratio $\frac{\Gamma(\pi^0, \pi^+\pi^-)}{\Gamma(\pi^0, \pi^0\eta)} = 18\%$ found by BES we need $\alpha = 0.54$

$\Rightarrow$ massive isospin violation difficult to justify

But there is a stronger reason to reject this value of $\alpha$...
Is the $\eta(1405)$ an $I=0$ object?

\[ \frac{d\Gamma}{dm_f} \text{ for } \eta' \rightarrow \pi^0 \pi^+ \pi^- \text{ decay in the } f_0(980) \text{ region} \]

$f_0(980)$ produced with its natural width $\sim 20$ MeV in disagreement with BES results

\[ \frac{d\Gamma}{dm_f} \text{ for } \eta' \rightarrow \pi^0 \pi^0 \eta \text{ decay in the } a_0(980) \text{ region} \]

ordinary shape

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Primary production vertex with the $K^*\bar{K}$ singularity


![Diagram of the primary production vertex with the $K^*\bar{K}$ singularity](image)

**novelty:** 2 singularity cuts in the loop function $G$ ($K^* \bar{K}$ and $K\bar{K}$)

$$G = i \int \frac{d^4q}{(2\pi)^4} \frac{\text{NUM}}{(p_\pi - q)^2 - m_{K^*}^2 + i\epsilon} \frac{1}{q^2 - m_{K}^2 + i\epsilon} \frac{1}{(P - q)^2 - m_{K}^2 + i\epsilon}$$

$\rightarrow$ superficially divergent

**problem:** the ratio $\frac{\Gamma(\pi^0, \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)}$ depends on an unknown form factor

$\rightarrow$ naturally solved in our approach
Primary production vertex with the $K^*\bar{K}$ singularity

regularized by a 3-momentum cutoff from meson-meson scattering data

$\rightarrow$ necessary choice in the background of chiral unitary approach

$\rightarrow$ the cutoff appears automatically in the loop function from the $K\bar{K} \rightarrow PP$ potential

$$V(\vec{q}, \vec{q}') = v\theta(q_{\text{max}} - |\vec{q}|)\theta(q_{\text{max}} - |\vec{q}'|) \ (\text{for s-waves})$$

it can be shown that, after the integration in $q^0$

$$G = \int_{|\vec{q}| < q_{\text{max}}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega} \frac{1}{p^0} \frac{1}{2\omega_{K^*}} \left[ \frac{\text{NUM}(q^0 = -\omega)}{p^0 - \omega - \omega_{K^*}} \frac{1}{p^0 + 2\omega} + \frac{\text{NUM}(q^0 = p^0 - \omega)}{p^0 - 2\omega + i\epsilon} \frac{1}{p^0 + p^0 - \omega - \omega_{K^*} + i\epsilon} \right]$$

$\implies$ logarithmically divergent

- $\eta(1405)$ considered as an $I = 0$ object
- $K^+K^-$ and $K^0\bar{K}^0$ channels appear with opposite sign

$$\implies t_f = G_{K^+K^*+} T_{K^+K^-,f} - G_{K^0K^*0} T_{K^0\bar{K}^0,f}$$
Results with the triangular diagram

\[ \frac{d\Gamma}{dm_f} \text{ for } \eta' \rightarrow \pi^0 \pi^+ \pi^- \text{ decay in the } f_0(980) \text{ region} \]

\[ \frac{d\Gamma}{dm_f} \text{ for } \eta' \rightarrow \pi^0 \pi^0 \eta \text{ decay in the } a_0(980) \text{ region} \]

the shapes are similar to the previous case but...

their ratio is much bigger!
Results with the triangular diagram

\[ \text{Ratio} \left( \frac{d\Gamma}{dm_f} \right)_{\pi^+\pi^-} / \left( \frac{d\Gamma}{dm_f} \right)_{\pi^0\eta} \] as a function of \( m_f \)

\[ \frac{\Gamma(\pi^0, \pi^+\pi^-)}{\Gamma(\pi^0, \pi^0\eta)} \simeq 16\% \] [including corrections]

closer to the experimental value of \( (17.9 \pm 4.2)\% \)

increase of one order of magnitude

\[ \implies \text{consequence of the two singularities in the triangle diagram} \]

(peculiar to the \( \eta(1405) \) case)
The $\eta(1475)$ and $\eta(1295)$

- In BES experiment [M. Ablikim et al. [BES III Collaboration], Phys. Rev. D 83, 032003 (2011)]

  >>> $\eta(1405)$ and $\eta(1475)$ indistinguishable

  >>> we evaluate the same ratio for the $\eta(1475)$:

  \[ \frac{\Gamma(\pi^0, \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)} \simeq 16\% \]

  >>> same result as before

- For the $\eta(1295)$ we get:

  - In the case of CPV

    \[ \frac{\Gamma(\pi^0, \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)} \simeq 1.7\% \]

  - In the case of $K^* \bar{K}$ production

    \[ \frac{\Gamma(\pi^0, \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)} \simeq 12\% \]

    (due to the fact that the $K^* \bar{K}$ channel is not open but close by)

  >>> the comparison with the experiment can give us informations about the $s\bar{s}$ component in the $\eta(1295)$
Conclusions

- the isospin violation is tied to the difference of masses between charged and neutral kaons
  \[ f_0 \text{ produced with } \Gamma = 9 \text{ MeV} \]

- assuming the primary $\pi^0PP$ production given by a contact term the value of \( \frac{\Gamma(\pi^0, \pi^+ \pi^-)}{\Gamma(\pi^0, \pi^0 \eta)} \) is too small compared to the one found by experiment

- following the approach of Wu et al. the ratio is increased of one order of magnitude
  - relating the cutoff in the new loop with the one from meson-meson scattering we can make a precise determination of the ratio
  - we get results very close to the one found by BES

- the present results strengthen the support for the $a_0(980)$ and $f_0(980)$ as dynamically generated
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